Quiz 8.1, 8.2

1 Problem 1

1. Calculate $\int x^2 \sin x \, dx$.

Well, we'll use the tabular method for this with $u = x^2$, $dv = \sin x \, dx$, taking derivatives down the u-column and integrating down the dv-column.

u	dv
x^2	$\sin x$
$2x$	$-\cos x$
2	$-\sin x$
0	$\cos x$

Then, multiplying u with v and alternating sign going down the list: $\int x^2 \sin x \, dx = +x^2(-\cos x) - 2x(-\sin x) + 2(\cos(x)) + C = (2 - x^2)\cos x + 2x \sin x + C$

2 OR Problem 2

2. Calculate $\int e^x \sin x \, dx$

We'll do this by integration by parts. Using LIPET, we set $u = e^x$, $dv = \sin x \, dx$. Then $du = e^x \, dx$, $v = -\cos x$:

$$\int e^x \sin x \, dx = e^x(-\cos x) - \int (-\cos x)e^x \, dx = \int e^x \cos x \, dx - e^x \cos x$$

Now, to find $\int e^x \cos x \, dx$, set $u = e^x$ again, so $dv = \cos x \, dx$. Then $v = \sin x$, so

$$\int e^x \cos x \, dx = e^x \sin x - \int e^x \sin x \, dx$$
$$\implies \int e^x \sin x \, dx = e^x \sin x - \int e^x \sin x \, dx - e^x \cos x$$

Adding $I = \int e^x \sin x \, dx$ to both sides yields $2I = e^x(\sin x - \cos x)$ so $I = e^x(\sin x - \cos x)/2 + C$.

3 OR Problem 3

3. Calculate $\int \tan^3 x \, dx$

There's two ways: turn everything into $\sin x$, $\cos x$ and split off a $\sin x$ to be $du = \sin x \, dx$ in the numerator, for a $u = \cos x \, dx$ u-sub, changing the remaining $\sin^2 x$ into $1 - \cos^2 x$:

$$\int \frac{\sin^3 x}{\cos^3 x} \, dx = \int \frac{\sin^2 x}{\cos^3 x} \sin x \, dx \quad (1)$$
$$= \int \frac{1 - \cos^2 x}{\cos^3 x} \sin x \, dx \quad (2)$$

Let $u = \cos x, du = -\sin x\, dx$, so $-du = \sin x\, dx$. Then:

$$\int \frac{1-\cos^2 x}{\cos^3 x} \sin x\, dx = \int (1-u^2)/u^3 - du \tag{3}$$
$$= \int u^{-1} - u^{-3}\, du \tag{4}$$
$$= \ln|u| - (\frac{u^{-2}}{-2}) + C_0 = \ln|\cos x| + \sec^2(x)/2 + C_0 \tag{5}$$

The other way is subbing $\tan^2 x = \sec^2 x - 1$, so $\int \tan^3 x\, dx = \int \tan^2 x \tan x\, dx = \int (\sec^2 x - 1)\tan x\, dx$. (why do these answers agree?) Then,

$$\int (\sec^2 x - 1)\tan x\, dx = \int \tan x \sec^2 x\, dx - \int \tan x\, dx = \int u\, du - (-\ln|\cos x|) + C_1 \tag{6}$$
$$= (\tan^2 x)/2 + \ln|\cos x| + C_1 \tag{7}$$

4 OR Problem 4

4. Calculate $\int \tan^4(1-y)\sec^4(1-y)\, dx$

First we'll do a u-sub: $u = 1-y$, $du = -dy$, so $dy = -du$. Then

$$\int \tan^4(1-y)\sec^4(1-y)\, dx = -\int \tan^4 u \sec^4 u\, du$$

To calculate -$\int \tan^4 u \sec^4 u\, du$, we'll split off a $\sec^2 u$ since there's an even number of secants:

$$-\int \tan^4 u \sec^4 u\, du = -\int \tan^4 u \sec^2 u \sec^2 u\, du \tag{8}$$
$$= -\int \tan^4 u(\tan^2 u + 1)\sec^2 u\, du \tag{9}$$

Now let $w = \tan u$ so $dw = \sec^2 u\, du$:

$$-\int \tan^4 u(\tan^2 u + 1)\sec^2 u\, du = -\int w^4(w^2+1)\, dw \tag{10}$$
$$= -\int w^6 + w^4\, dw \tag{11}$$
$$= -w^7/7 - w^5/5 + C = -\tan^7 u/7 - \tan^5 u/5 + C \tag{12}$$
$$= -(\tan^7(1-y))/7 - (\tan^5(1-y))/5 + C \tag{13}$$